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GEOMETRY AND PHYSICS

Cosmin-Constantin NIȚU

University of Agronomic Sciences and Veterinary Medicine of Bucharest,
59 Marasti Blvd, District 1, Bucharest, Romania

Corresponding author email: cosmin.nitu@fifim.ro

Abstract

All science is connected and the main connection is through the language of mathematics. Some of the most spectacular scientific results are obtained transdisciplinary. The relationship between geometry and physics is one of the most profound in science, evolving from a tool for measurement into the very language used to describe the fundamental laws of the universe. Geometry and physics are deeply intertwined disciplines, where geometry provides the formal language and structural framework for physical laws, while physics often drives the development of new mathematical theories. Differential geometry, for example, is essential for understanding gauge theories in particle physics and Riemannian geometry is fundamental in cosmology. Other branches of mathematics, such topology and Lie groups contribute to both classical and modern physics. In this article we investigate some geometric results obtained via physical methods and we emphasize some other links between geometry and physics.

Key words*: geometry, physics, string theory, theory of relativity, vectors.*

"As far as the laws of mathematics refer to reality, they are not certain; and as far as they are certain, they do not refer to reality"- Albert Einstein, 1921

INTRODUCTION

The deep and enduring relationship between geometry and physics has played a central role in the development of modern scientific thought. From the early formulation of classical mechanics to the emergence of contemporary theories of spacetime, geometric concepts have provided a powerful language for describing the fundamental structure of physical reality. In particular, the use of mathematical frameworks such as Euclidean and non-Euclidean geometry has enabled physicists to model space, motion, and interactions with increasing precision and abstraction.

In classical physics, geometry was primarily viewed as a static and absolute stage upon which physical processes unfold. However, this perspective underwent a profound transformation with the advent of modern physics. The development of theories such as general relativity demonstrated that geometry is not merely a passive background, but an active and dynamic component of the physical universe. In this context, the curvature of spacetime becomes directly related to the distribution of mass and energy, illustrating a deep interplay between geometric structures and physical phenomena.

Geometry lies at the very heart of relativity, transforming it from a theory about forces into one about the structure of spacetime itself. In special relativity, the geometry is that of Minkowski space, where time and space are unified into a four-dimensional continuum with a non-Euclidean metric that determines invariant intervals between events. General relativity deepens this connection by describing gravity not as a force, but as the curvature of spacetime induced by mass and energy. The mathematical language of differential geometry-manifolds, metrics, geodesics, and curvature tensors-provides the framework for this description. Objects move along geodesics, which generalize the notion of straight lines to curved spacetime, and the Einstein field equations relate the geometric properties of spacetime directly to the physical distribution of matter and energy.

Geometry plays a central and indispensable role in string theory, where the fundamental objects are not point particles but one-dimensional strings whose vibrational modes depend sensitively on the geometry of the space in

which they propagate. In this framework, spacetime is typically extended beyond four dimensions, and the extra dimensions are assumed to form compact geometric spaces, often modelled by Calabi-Yau manifolds. The specific shape and topology of these compact spaces determine crucial physical properties, such as particle spectra and interaction strengths. Moreover, concepts from modern geometry-such as complex geometry, topology, and mirror symmetry-are deeply intertwined with the structure of string theory, revealing surprising dualities between seemingly distinct physical models. In this way, geometry is not merely a background setting, but an active ingredient that shapes the physical content of the theory itself (Steiner, 1992; Bailly et al., 2011; Rédei, 2020). This paper aims to explore fundamental connections between classical geometry and physics, highlighting how geometric methods contribute to our understanding of physical laws. By examining key theoretical frameworks and their applications, we seek to demonstrate that geometry is not only a mathematical tool, but also an essential foundation for the formulation and interpretation of physical theories.

MATERIALS AND METHODS

I. Vectors in geometry

In mathematics and physics, vectors are the same, as numerical support, but in physics they have an interpretation in some phenomena.

From the algebraic point of view, a *vector* is an element $x = (x_1, x_2, \dots, x_n) \in \mathbb{R}^n$. Vectors can naturally be added, substracted and multiplied by a scalar, in the same way as matrices, so $\mathbb{R}^n$ has a natural structure of vectorial space.

From the geometric point of view, a vector (Figure 1) is an "arrow" between two points (Berger, 1987).

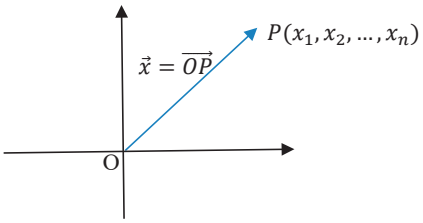


Figure 1. Vector

The Euclidian *scalar product* of two vectors $x, y \in \mathbb{R}^n$ is defined by the equality

$$x \cdot y = x_1 y_1 + x_2 y_2 + \cdots x_n y_n.$$

The Euclidian *norm* of a vector is

$$\|x\| = \sqrt{x_1^2 + x_2^2 + \cdots + x_n^2}.$$

Remark 1. $x^2 = x \cdot x = \|x\|^2$.

From *Cauchy-Buniakovski-Schwarz* inequality we have

$$|x \cdot y| \leq \|x\| \cdot \|y\|.$$

Thus, the angle between the two vectors is defined by

$$\cos\theta = \frac{x \cdot y}{\|x\| \cdot \|y\|} \in [-1, 1].$$

Remark 2. $x \perp y \Leftrightarrow x \cdot y = 0$.

II. Mass centers

It is well known that the mass center G of system of material points in plane, of coordinates $P_i(x_i, y_i)$ in which there are concentrated the weights $m_i \geq 0,\ i = \overline{1,n}$ is given by (Solomentsev, 2001),

$$x_G = \frac{m_1 x_1 + m_2 x_2 + \cdots m_n x_n}{m_1 + m_2 + \cdots m_n}$$

and

$$y_G = \frac{m_1 y_1 + m_2 y_2 + \cdots m_n y_n}{m_1 + m_2 + \cdots m_n}$$

Therefore, considering the masses $m_i = m\ (constant)$ in the vertices of a triangle ABC, the mass center of the system

$$S = \{A(m), B(m), C(m)\}$$

has the same coordinates of the geometric mass center of the triangle ABC, which is the intersection of its medians:

$$x_G = \frac{x_A + x_B + x_C}{3}$$

and

$$y_G = \frac{y_A + y_B + y_C}{3}$$

The mass center of the plate ABC is $G(x_G, y_G)$, where.

$$x_G = \frac{\iint_{ABC} x\, \rho(x,y)\, dxdy}{\mu}$$

and

$$y_G = \frac{\iint_{ABC} y\,\rho(x,y)dxdy}{\mu}$$

where the mass of the plate ABC is

$$\mu = \iint_{ABC} \rho(x,y)\,dxdy.$$

Definition 1. We consider the points $P_i(x_i, y_i) \in \mathbb{R}^2$, forming a polygon $P_1P_2 \dots P_n$. If

$$x_m = \min_{i=\overline{1,n}} x_i\ ,\ x_M = \max_{i=\overline{1,n}} x_i,$$

$$y_m = \min_{i=\overline{1,n}} y_i \text{ and } y_M = \max_{i=\overline{1,n}} y_i$$

then we call the rectangle

$$D_{P_1P_2\dots P_n} = [x_m, x_M] \times [y_m, y_M]$$

the ***rectangle cover*** of the polygon $P_1P_2 \dots P_n$.

APPLICATIONS

I. Applications of physics in geometry

Theorem of Pythagoras (Figure 2)

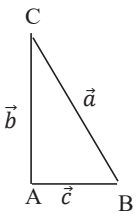


Figure 2. Rectangle triangle

By denoting $\overrightarrow{AB} = \vec{c},\ \overrightarrow{CB} = \vec{a}$ and $\overrightarrow{CA} = \vec{b}$, and $AB = c,\ BC = a\ and\ AC = b$ we have

$$\vec{b} + \vec{c} = \vec{a}\Big|^2$$

$$\vec{b}^2 + 2\underbrace{\vec{b}\cdot\vec{c}}_{0} + \vec{c}^2 = \vec{a}^2$$

$$\left\|\vec{b}\right\|^2 + \|\vec{c}\|^2 = \|\vec{a}\|^2$$

Thus, we obtain the theorem of Pythagoras.

$$b^2 + c^2 = a^2$$

Stewart's relation (Figure 3)

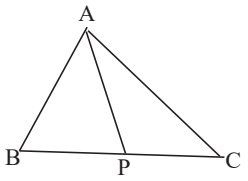


Figure 3. A scalene triangle with a cevian

By denoting $\overrightarrow{AB} = \vec{c},\ \overrightarrow{CB} = \vec{a}$ and $\overrightarrow{CA} = \vec{b}$, $\overrightarrow{AP} = \vec{p}$, and $AB = c, BC = a\ and\ AC = b$.

If $\frac{BP}{BC} = k \in (0,1)$, then $\frac{PC}{BC} = 1 - k$

$$\vec{c} = \vec{p} + k\vec{a}$$

$$\vec{b} = \vec{p} + (k-1)\vec{a}.$$

By raising the above equalities to square, we obtain

$$\vec{c}^2 = \vec{p}^2 + k^2\vec{a}^2 + 2k\vec{a}\vec{p}\ |\cdot(1-k)$$

$$\vec{b}^2 = \vec{p}^2 + (1-k)^2\vec{a}^2 + 2(k-1)\vec{a}\vec{p}\ |\cdot k$$

After the multiplication, we add the relation and it results an equivalent to Stewart's theorem:

$$AB^2 \cdot PC + AC^2 \cdot PB - BC^2 \cdot PA = PA \cdot PB \cdot PC$$

Theorem 1. Let ABC be a triangle with all the angles smaller than 120°. Then the point P in the plane of the triangle, for which the sum

$$PA + PB + PC = minimum$$

is the one which has the property that

$\widehat{APB} = \widehat{APC} = \widehat{BPC} = 120^o$, denoted by T
(the *Torricelli point* of the triangle - Figure 4)

Proof. We consider in the plane of the triangle ABC, assumed parallel to the ground, 3 threads tied at a point P, equal in length, passing through the points A, B and C and having the same weight m at the other end. The forces $\overrightarrow{F_1}, \overrightarrow{F_2}$ and $\overrightarrow{F_3}$ that act on the point P are equal in module to the gravitational forces $\overrightarrow{G_1}, \overrightarrow{G_2}$ and $\overrightarrow{G_3}$.

This system of forces reaches the equilibrium state when its potential gravitational energy is minimum. If we denote by h_1, h_2 and h_3 the distances from the weights to the ground then the potential gravitational energy is

$$E_p = mgh_1 + mgh_2 + mgh_3$$
$$= mg(h_1 + h_2 + h_3)$$

Therefore, the sum

$h_1 + h_2 + h_3$ is minimum,

which implies that

$PA + PB + PC$ is minimum.

Also, at equilibrium

$$\overrightarrow{F_1} + \overrightarrow{F_2} + \overrightarrow{F_3} = \vec{0}$$

which proves that the angles between these forces are all equal with 120^o and ends the proof (Nicolescu & Boskoff, 1990).

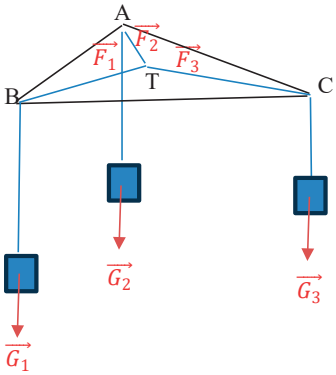


Figure 4. Torricelli's point

II. Aplications of geometry in physics

Theorem 2. Considering a fixed triangle ABC in a plane, and the variable masses $m_1, m_2, m_3 \geq 0$, not all nulls, the mass center of the system $S = \{A(m_1), B(m_2), C(m_3)\}$ can be any point of the rectangle D_{ABC} (Figure 5).

Proof.

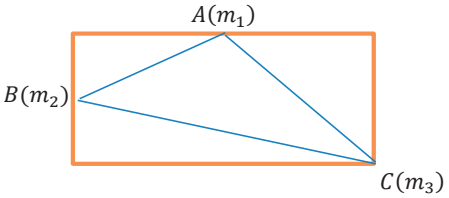


Figure 5. The rectangle cover of a triangle

We consider the points

$$A(x_1, y_1), B(x_2, y_2), C(x_3, y_3) .$$

Without loss of generality, we can assume that

$$m_1 + m_2 + m_3 = 1.$$

Otherwise, we can replace them with

$$m_i' = \frac{m_i}{m_1+m_2+m_3},$$

which have the property

$$m_1' + m_2' + m_3' = 1.$$

We observe that the mass center of the system $S = \{A(m_1), B(m_2), C(m_3)\}$ verifies the inequalities:

$$x_G = \frac{m_1x_1 + m_2x_2 + m_3x_3}{m_1 + m_2 + m_3} = m_1x_1 + m_2x_2 + m_3x_3 \geq (m_1 + m_2 + m_3)x_m = x_m$$

and also

$$x_G = \frac{m_1x_1 + m_2x_2 + m_3x_3}{m_1 + m_2 + m_3} = m_1x_1 + m_2x_2 + m_3x_3 \leq (m_1 + m_2 + m_3)x_M = x_M$$

Thus $x_G \in [x_m, x_M]$ and in the same way it can be shown that $y_G \in [y_m, x_M]$.

So

$$G(x_G, y_G) \in [x_m, x_M] \times [y_m, y_M] = D_{ABC}.$$

Now we show that any point $(a, b) \in D_{ABC}$ is the mass center of a unique system $S = \{A(m_1), B(m_2), C(m_3)\}$, with $m_1 + m_2 + m_3 = 1$.

In this scope, we have to prove that the system of linear equations:

$$\begin{cases} m_1x_1 + m_2x_2 + m_3x_3 = a \\ m_1y_1 + m_2y_2 + m_3y_3 = b \\ m_1 + m_2 + m_3 = 1 \end{cases}$$

has a unique solution

$$(m_1, m_2, m_3).$$

This is true, by Cramer's rule, because the determinant of the system's matrix is

$$\Delta = \begin{vmatrix} x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \\ 1 & 1 & 1 \end{vmatrix}$$

and $\Delta \neq 0$, as $Area\,(ABC) = \frac{1}{2}|\Delta| \neq 0$, which ends the proof.

Theorem 3. Considering a fixed triangle ABC in a plane, with the sides [AB], [AC] and [BC] made of homogenous materials of masses $\mu_1, \mu_2, \mu_3 \geq 0$, the mass center of the the triangle ABC can be any point of D_{MNP}, where M, N and P are the middle points of the sides [AB], [AC] and [BC].

Proof. From the physical point of view, the triangle ABC can be viewed as mass equivalent to the the system $S = \{M(\mu_1), N(\mu_2), P(\mu_3)\}$ and, by applying Theorem 1, we obtain the desired result.

Theorem 4. Let be a triangular plate ABC, where A, B and C are fixed points in a plane and we assume that the density function $\rho: ABC \to \mathbb{R}_+$ is integrable. Then, any point of D_{ABC} can be the mass center of such a triangle plate.

Proof. The mass center of the plate ABC is $G(x_G, y_G)$, where

$$x_G = \frac{\iint_{ABC} x\,\rho(x,y)dxdy}{\mu}$$

$$y_G = \frac{\iint_{ABC} y\,\rho(x,y)dxdy}{\mu}$$

and the mass of the plate ABC is

$$\mu = \iint_{ABC} \rho(x,y)\,dxdy.$$

The triangle ABC, along with its interior, is a compact set in $\mathbb{R}^2$, as it is closed and bounded and we consider the projections

$x_m = \min pr_{Ox}ABC$, $x_M = \max pr_{Ox}ABC$, and

$y_m = \min pr_{Oy}ABC$, $y_M = \max pr_{Oy}ABC$

We have the inequalities:

$$x_m \le x \le x_M, \forall\, x \in \text{ABC}$$

Therefore, by integrating, we obtain

$$x_m = \frac{x_m \iint_{ABC} \rho(x,y)\,dxdy}{\mu} \le x_G$$

$$\le \frac{x_M \iint_{ABC} \rho(x,y)\,dxdy}{\mu} = x_m$$

Analogue,

$$y_m \le y_G \le x_M.$$

Now, for a fixed point $(a,b) \in D_{ABC}$, we consider the case from the Theorem 1, which ends the proof.

CONCLUSIONS

The interplay between geometry and physics reveals a profound unity between mathematical abstraction and the description of the natural world. The results presented in this paper reinforce the idea that geometric frameworks are not merely auxiliary tools, but essential components in the formulation of physical theories. Through the analysis conducted, it becomes evident that physical phenomena can often be more deeply understood when interpreted through a geometric lens.

Moreover, the continued development of modern physics suggests that geometry will remain a fundamental pillar in future theoretical advancements. As new models and approaches emerge, the refinement of geometric methods will likely play a crucial role in addressing unresolved questions and extending the boundaries of current knowledge (Jost, 2009; Landi et al., 2017).

Ultimately, this article highlights the enduring significance of geometry in physics, emphasizing its capacity to unify concepts, clarify complex relationships, and provide insight into the understanding the structure of reality, because physics and geometry are inseparable.

REFERENCES

Bailly, F., & Longo, G. (2011). *Mathematics and the natural sciences: the physical singularity of life.* World Scientific.

Berger, M. (1987). *Geometry I*, Springer Berlin, Heidelberg.

Einstein, A. (1921). *Geometry and Experience* (lecture).

Jost, J. (2009), *Geometry and Physics*, Springer Berlin, Heidelberg.

Nicolescu, L., & Boskoff, V. (1990). *Probleme practice de geometrie (Practical problems of geometry)*, Ed. Tehnică.

Rédei, M. (2020). On the tension between physics and mathematics, *J Gen Philos Sci 51*, pp. 411–425.

Solomentsev, E.D. (2001). Euclidean space. In *Encyclopedia of Mathematics,* EMS Press. encyclopediaofmath.org.

Landi, G., Rubtsov, V., & Sala, F. (2017). Interactions between Geometry and Physics: A Special Issue in Honor of Ugo Bruzzo's 60th Birthday. *Journal of Geometry and Physics*, 118, 1–3.

Steiner, M. (1992). Mathematical Rigor in Physics. In M. Detlefsen (Ed.), *Proof and Knowledge in Mathematics* (pp. 158–170). Routledge.